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Induced gravity on intersecting branes

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Abstract: We establish Einstein-Hilbert gravity couplings in the effective action for Intersecting Brane Worlds. The four-dimensional induced Planck mass is determined by calculating graviton scattering amplitudes at one-loop in the string perturbation expansion. We derive a general formula linking the induced Planck mass for $\mathcal{N} = 1$ supersymmetric backgrounds directly to the string partition function. We carry out the computation explicitly for simple examples, obtaining analytic expressions.

Keywords: Superstrings and Heterotic Strings, D-branes.

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1. Introduction

Intersecting Brane Worlds are among the most promising candidates for constructing consistent string theory vacua with a realistic low-energy limit. They are usually constructed from D6-branes which intersect on a four-manifold and wrap different three-cycles in the transverse compact space [1, 2] (for recent reviews and more references see [3, 4]). Open strings stretching from one brane to another give rise to chiral fermions which are localized on the intersection locus due to the finite string tension. Interactions of the open string fields have been studied in some detail, for instance with respect to gauge coupling threshold corrections [5, 6], Yukawa couplings [7]–[10] and the tachyon potential [11]–[14]. On the other hand, gravitational couplings have received much less attention.[1] It is clear that knowing their exact form will be crucial in determining the cosmological evolution of Intersecting Brane Worlds and also for studying low energy phenomenology. The simplest non-trivial gravity coupling is an induced Einstein-Hilbert term on the intersection locus. We demonstrate that such a term arises at one-loop level in the string perturbation series and show how to compute its magnitude. Although in this article we are only concerned with the Einstein-Hilbert term, one-loop computations like ours will have other important applications. For example, in the context of moduli stabilization, it will be interesting to determine one-loop corrections to the scalar potential in string compactifications.

[1] Interaction terms for the geometric moduli of toroidal type IIB orientifolds have recently been computed in [15].

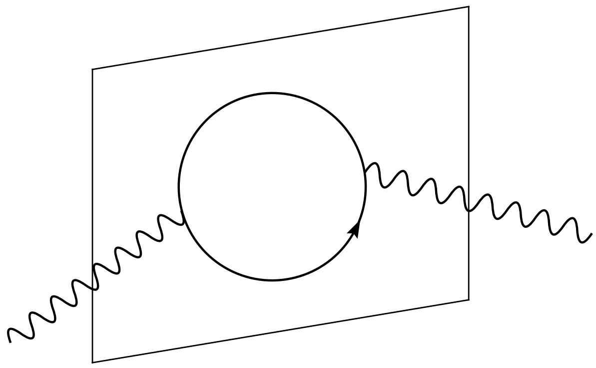

Figure 1: Localized matter loops on a surface Σ renormalize the graviton two-point function.

Our interest in induced Einstein gravity is partly motivated by the observation by Dvali, Gabadadze and Porrati (DGP) [16, 17] that an effective gravity action of the form

$$S = M^{D-2} \int d^D x \sqrt{-g}\mathcal{R}^{(D)} + M_P^2 \int_\Sigma d^4 x \sqrt{-\tilde{g}}\mathcal{R}^{(4)} \tag{1.1}$$

leads to localized four dimensional gravity on the subspace Σ of a flat D-dimensional bulk, irrespective of the actual volume of the extra dimensions. The first term in the above expression is the Einstein-Hilbert action for the bulk which is constructed from the D-dimensional metric g whereas the second term is the Einstein-Hilbert on Σ which is constructed from the pullback metric $\tilde{g}$. For an infinitely thin subspace[2] Σ, the localization was shown to be perfect if $D \geq 6$. On the other hand, if one assumes an effective width for the subspace in question, the bulk space becomes infrared transparent leading to D-dimensional gravity at ultra-low energies. This is in sharp contrast to the Kaluza-Klein scenario where extra dimensions become visible at high energies.

The assumptions leading to the DGP model are in fact extremely generic. An Einstein-Hilbert term on a subspace Σ can easily be generated from interactions of localized matter with bulk gravitons. From a field theory perspective, the graviton propagator receives corrections from diagrams with matter on the brane running in loops. In a purely four-dimensional spacetime, these induce an Einstein-Hilbert term whenever the matter fields are non-conformal [18]. Therefore, we can expect a non-vanishing four-dimensional Planck mass in all but the most special circumstances. In order to realize DGP's scenario in string theory, one needs to find configurations where localized matter leads to an induced Einstein-Hilbert action. The simplest setup with localized matter is a single type II D-brane for which the gravitational couplings have been computed at tree level in reference [19]. In this particular case the large supersymmetry prevents an Einstein-Hilbert term and the induced gravity action starts with terms which are quadratic in curvature. Similarly, a single orientifold plane - on which twisted closed strings are localized - does not lead to DGP-gravity (see [20] for a tree-level computation of the effective action). On the other hand, string

[2]We deliberately avoid the term brane because in the context of this paper, the subspace Σ will be the intersection locus of branes at angles

theory realizations of induced gravity have been found for a variety of backgrounds [22]. Orbifolds [21, 22] and, more generally, Calabi-Yau compactifications [23]–[25] have been considered as well as non-compact orientifolds of type IIB string theory [26]–[28]. From a phenomenological point of view, the latter are interesting because the requirement of a crystallographical action of the orbifold group can be dropped and one has more freedom in constructing GUT models. For the specific case of non-compact $\mathbb{Z}_N$ orientifolds with $D3$-branes, the induced four-dimensional Planck mass has been obtained explicitly [28] from a string theory computation.

The purpose of this paper is to establish induced gravity on intersecting branes. Indeed, the low-energy effective action for intersecting brane worlds generically has running gauge couplings and is therefore non-conformal. Thus, from a four-dimensional point of view, we can expect to find an induced Einstein-Hilbert term on the intersection locus. In this paper, we only consider intersecting brane worlds with at least $\mathcal{N}=1$ supersymmetry in four dimensions because these provide the only known examples in which both RR- and NSNS-tadpoles are cancelled, enabling us to perform one-loop computations. In section 2, we show how the induced Planck mass can be computed in this class of models. In section 3, we explicitly carry out the computation for supersymmetric $\mathbb{Z}_N$ orientifolds of T^6 with local tadpole cancellation. The results we obtain are simple analytical expressions. The induced four-dimensional Planck mass generically receives contributions which are independent of the compactification scale R. Curiously, we find that for even order orbifold groups, the induced Planck mass additionally receives corrections of the order R^2 if one of the internal tori is taken to be large. These can be interpreted as coming from a Kaluza-Klein reduction of a six-dimensional induced Planck mass. In section 4, we discuss our results.

2. Graviton scattering

We obtain the effective gravity action using the S-matrix approach, i.e. by computing the amplitude for a graviton scattering off the brane intersection locus and comparing the result to a field theory ansatz. Since we are interested in the effective field theory on the worldvolume of the brane intersection we have to use string worldsheets which are localized correspondingly. The lowest order worldsheet which satisfies this requirement is the annulus which has its two boundaries attached to two different branes. We obtain the scattering amplitude by inserting two graviton vertices in the path integral. However, the annulus alone gives a divergent expression for the scattering amplitude. This is because the D-branes carry tadpoles which lead to divergences when the corresponding fields propagate in the closed string channel. At one-loop level, it is therefore impossible to consider two intersecting branes independently from the background they are in. Instead, we always have to work with the full set of one-loop surfaces. For string vacua with compact extra dimensions, uncancelled RR-tadpoles imply inconsistent equations of motion for the RR p-forms [29]. Thus, their cancellation is a necessary consistency check for any such open string vacuum. On the other hand, NS-NS tadpoles merely hint at a perturbative instability of the chosen vacuum. Thus, for some time they had been largely ignored in D-brane model building. However, any perturbative loop computation is rendered meaningless if there are uncancelled tadpoles, of any kind, and when computing loop corrections to scattering amplitudes, we are forced to consider only perturbatively consistent and stable backgrounds.

The only intersecting brane worlds with complete tadpole cancellation we know of are supersymmetric orientifolds [2] and [30]–[32]. Just like for the vacuum amplitude, divergencies in the graviton scattering amplitude should be cancelled between the annulus, Klein bottle and Möbius strip. Also, from the orbifold section we have twisted closed strings which contribute to the induced Einstein-Hilbert term through the torus worldsheet. The details of the string computation are similar to those presented in [28] where induced gravity in standard orientifolds with $D3$-branes was investigated, i.e. in orientifolds with projection ΩJ where J acts on the complexified coordinates of the extra dimensions as $J : Z_i \mapsto -Z_i$. Contrastingly, orientifolds with intersecting branes arise from an orientifold projection $\Omega\mathcal{R}$ where $\mathcal{R}$ acts as $\mathcal{R} : Z_i \mapsto \overline{Z}_i$. The torus amplitude is unaffected by the orientifold projection. Hence, the result that the torus contribution to the four-dimensional Einstein-Hilbert term is proportional to the Euler number of the compactification space [23] carries over to orientifold models with intersecting branes. The difference between orientifolds with intersecting branes and the models presented in [28] is what might be called the orientifold sector, i.e. the contributions from the Klein bottle, annulus and Möbius strip. We find that this difference leads to considerable complications. In particular, it is not always obvious how (if at all) tadpole cancellation guarantees the absence of UV-divergences in the graviton scattering amplitude. However, for the simplest case in which tadpoles are cancelled locally by placing an appropriate number of D-branes on top of the orientifold planes, we can carry out the computation explicitly and obtain a finite answer. First, we will outline the general procedure for computing the relevant scattering amplitudes with special emphasis on correctly normalizing the path integral and apply our results to concrete models in section 3.

2.1 Vertex operators, propagators and path integral normalization

We are interested in contributions to the graviton scattering amplitude coming from the annulus ($\mathcal{A}$), Klein bottle ($\mathcal{K}$) and Möbius strip ($\mathcal{M}$). In the path integral approach, they are given by:

$$A_\sigma = \int d^2z_1 \int d^2z_2 \langle V_{p_1,\varepsilon_1}(z_1,\overline{z}_1) V_{p_2,\varepsilon_2}(z_2,\overline{z}_2)\rangle_\sigma\,; \qquad \sigma = \mathcal{A},\mathcal{K},\mathcal{M}\,, \tag{2.1}$$

where $\langle\ldots\rangle_\sigma$ denotes the Polyakov path integral for the NSR string on a Riemann surface σ. The path integral includes summation over different spin structures s and metric moduli t. As we are dealing with a CP-even scattering process, only the even spin structures contribute to the amplitude and we can therefore use the graviton vertex in the $(0,0)$-picture:

$$V_{p,\varepsilon}(z,\overline{z}) = g_c \varepsilon_{\mu\nu} : (\partial X^\mu + i\psi^\mu p\cdot\psi)(z)\left(\overline{\partial} X^\nu - i\tilde{\psi}^\nu p\cdot\tilde{\psi}\right)(\overline{z}) e^{ip\cdot X(z,\overline{z})} : . \tag{2.2}$$

Here, g_c is the closed string coupling and we have set $\alpha' = 2$. Since we are interested in the overall magnitude of the graviton scattering amplitude, the correct normalization of the path integral is crucial. For the surfaces in question, it is unambiguously given by

$$\langle 1 \rangle_{\text{one-loop}} = \int_0^\infty \frac{dt}{2t} \operatorname{Tr}\left(P_{\text{orientifold}} P_{\text{GSO}} e^{-2\pi t \alpha' H} \right), \tag{2.3}$$

as can be checked by applying the Coleman-Weinberg formula [33] to the string theory particle content. Expanding the orientifold projection $P_{\text{orientifold}}$, one obtains the individual contributions from all four one-loop surfaces. The normalization of the vertex operator (2.2) is the same for all topologies and spin structures, it defines the string coupling [34].

All three surfaces $\mathcal{A}, \mathcal{K}$ and $\mathcal{M}$ in the orientifold sector can be obtained from a covering torus by an identification under an appropriate anti-holomorphic involution

$$I_{\mathcal{A}}, I_{\mathcal{K}} : z \to 1 - \overline{z}\,, \qquad I_{\mathcal{M}} : z \to 1 - \overline{z} + \frac{\tau}{2}\,. \tag{2.4}$$

The complex modulus τ of the covering torus is related to the loop modulus t via

$$\tau = \begin{cases} it/2 & \mathcal{A} \\ 2it & \mathcal{K} \\ 1/2 + it/2 & \mathcal{M}\,. \end{cases} \tag{2.5}$$

The propagators of the worldsheet fields X and ψ on a surface σ with a particular spin structure s can be derived from the corresponding expressions for the covering torus by using the method of images [36]. We write them as operator equations:

$$\begin{aligned}
\partial X^\mu(z) \partial X^\nu(w) &= \eta^{\mu\nu} \left(\partial_z \partial_w P_B(z, w; \tau) + \frac{\pi}{4\tau_2} \right) && (2.6) \\
\partial X^\mu(z) \overline{\partial} X^\nu(\overline{w}) &= \eta^{\mu\nu} \left(\partial_z \partial_{\overline{w}} P_B(z, I_\sigma(w); \tau) - \frac{\pi}{4\tau_2} \right) && (2.7) \\
\psi^\mu(z) \psi^\nu(w) &= \eta^{\mu\nu} P_F(z, w; \tau, s) && (2.8) \\
\psi^\mu(z) \tilde{\psi}^\nu(\overline{w}) &= \eta^{\mu\nu} P_F(z, I_\sigma(w); \tau, s) && (2.9) \\
\tilde{\psi}^\mu(\overline{z}) \tilde{\psi}^\nu(\overline{w}) &= \eta^{\mu\nu} \overline{P}_F(\overline{z}, \overline{w}; \tau, \overline{s})\,, && (2.10)
\end{aligned}$$

where $\sigma = \mathcal{A}$, $\mathcal{K}$, $\mathcal{M}$ denotes the surface and P_B, P_F are the bosonic and fermionic propagators on the torus

$$\begin{aligned}
P_B(z, w; \tau) &= -\frac{1}{4} \ln \left| \frac{\vartheta_1(z - w|\tau)}{\vartheta_1'(0|\tau)} \right|^2 + \frac{\pi (z_2 - w_2)^2}{2\tau_2} && (2.11) \\
P_F(z, w; \tau, s) &= \frac{i}{2} \frac{\vartheta_s(z - w|\tau)}{\vartheta_1(z - w|\tau)} \frac{\vartheta_1'(0|\tau)}{\vartheta_s(0|\tau)}\,, && (2.12)
\end{aligned}$$

involving Jacobi's theta functions (for definitions and selected properties, see appendix A). By 'operator equation' we mean that an equation holds inside a path integral. For instance, we have $\langle \psi^\mu(z) \psi^\nu(w) \rangle = \langle \eta^{\mu\nu} P_F(z, w; \tau, s) \rangle$. Note that we cannot simply drop the brackets on the right hand side because the Polyakov path integral is not normalized to unity,

$\langle 1 \rangle \neq 1$. There is another subtlety regarding the overall normalization of the scattering amplitude which stems from the fact that for compact worldsheets, the ordinary Green's functions for the worldsheet scalars don't exist. Instead, one has to exclude the zero-mode contributions. This doesn't make a difference for contractions of the form $\partial X \partial X$ but it matters for contractions between the exponentials in the graviton vertices. Because normal ordering is defined with respect to the oscillator modes, in evaluating the expression $\langle V_{p_1,\varepsilon_1} V_{p_2,\varepsilon_2} \rangle$, we get a factor of $\langle e^{i(p_1+p_2)x_0} \rangle$ in addition to all full contractions of the oscillator part of the world-sheet fields. Here, x_0^μ is the zero-mode of the worldsheet scalars. If we denote by $\langle \ldots \rangle_*$ the path integral with contributions from x_0^μ excluded ($\mu = 0 \ldots 3$), we find:

$$\langle e^{i(p_1+p_2)x_0} \rangle = \int d^4 x_0 e^{i(p_1+p_2)x_0} \langle 1 \rangle_* = (2\pi)^4 \delta^{(4)}(p_1+p_2) \langle 1 \rangle_* = (2\pi)^4 \delta^{(4)}(p_1+p_2) \frac{1}{V_4} \langle 1 \rangle \,, \tag{2.13}$$

where V_4 is the regulated four-dimensional volume. Using this identity, we can directly relate the two graviton scattering amplitude to the vacuum amplitude. Before doing so, we introduce one more piece of notation:

$$\langle 1 \rangle_\sigma = \sum_s \int_0^\infty \frac{dt}{2t} Z_\sigma(t,s) \,. \tag{2.14}$$

Also, for notational convenience, we will let a product of two fields stand for its contraction (propagator) whenever it appears outside the path integral. Our discussion of the path integral normalization also applies to the work in [28] where the normalization of the scattering amplitudes was left undetermined.

2.2 One-loop renormalization of the four-dimensional Planck mass

Now we have all the ingredients for the computation of the graviton scattering amplitude (2.1). Because we are interested in supersymmetric backgrounds, the one-loop vacuum amplitude vanishes when summed over spin structures. Therefore, we can only get a non-vanishing contribution to the scattering amplitude if we contract at least one pair of fermionic fields. Looking at the graviton vertex, we see that this already amounts to the correct power of momenta for the induced Einstein-Hilbert term[3] and therefore we find that, taking into account the physical state conditions for graviton vertices, the relevant contribution to the scattering amplitude is given by:

$$\begin{aligned} A = {} & (2\pi)^4 \delta^{(4)}(p_1+p_2) \frac{1}{V_4} (-g_c^2) \sum_{\sigma=\mathcal{A},\mathcal{K},\mathcal{M}} \sum_{s=2,3,4} \int_0^\infty \frac{dt}{2t} Z_\sigma(t,s) \int d^2 z_1 \int d^2 z_2 \, \times \\ & \times \frac{1}{4} \left\{ \partial X \partial X (\tilde{\psi}\tilde{\psi})_s^2 - \overline{\partial} X \partial X (\psi\tilde{\psi})_s^2 + \overline{\partial} X \overline{\partial} X (\psi\psi)_s^2 - \partial X \overline{\partial} X (\tilde{\psi}\psi)_s^2 \right\} (p_1 \varepsilon_2 \varepsilon_1 p_2) \,. \end{aligned} \tag{2.15}$$

We should mention that this expression actually vanishes because of four-dimensional momentum conservation and transversality of the graviton polarization. For a completely

[3]We also see that, as expected, the four-dimensional cosmological constant does not receive corrections at one-loop and we are indeed dealing with a DGP type of gravity.

strict computation, we would have to either turn on momentum components in the transverse directions (for which momentum conservation doesn't hold) or use the three graviton scattering amplitude. However, both options lead to considerably more complicated expressions. Instead of following these prescriptions, we simply use the structure of the kinetic term $(p_1\varepsilon_2\varepsilon_1 p_2)$ to identify the corresponding term in the effective field theory. In fact, it is the only possible combination at quadratic order in the momenta and is reproduced by the quadratic Einstein-Hilbert action on Minkowski four-space:

$$S_{EH} = \frac{M_P^2}{2}\int d^4x \left\{ -\frac{1}{2} h_{\alpha\beta,\nu} h^{\nu\beta,\alpha} \right\}. \tag{2.16}$$

Here, we have dropped terms in the action which vanish in transverse traceless gauge because these do not contribute to the tree level scattering amplitude. It is common practice to parameterize the one-loop renormalization of the Einstein-Hilbert term by a number δ which is related to the induced Planck mass by $\delta = M_{P,\mathrm{ind}}^2/2$. Thus, comparing the coefficients of the kinetic term, we find that the one-loop renormalization of the Planck-mass is given by

$$\begin{aligned}\delta = \frac{g_s^2}{V_4} \sum_{\sigma=\mathcal{A},\mathcal{K},\mathcal{M}} \sum_{s=2,3,4} \int_0^\infty \frac{dt}{2t} Z_\sigma(t,s) \int d^2 z_1 \int d^2 z_2 \times \\ \times \frac{1}{4}\left\{ \partial X \partial X (\tilde\psi\tilde\psi)_s^2 - \overline{\partial} X \partial X (\psi\tilde\psi)_s^2 + \overline{\partial} X \overline{\partial} X (\psi\psi)_s^2 - \partial X \overline{\partial} X (\tilde\psi\psi)_s^2 \right\},\end{aligned} \tag{2.17}$$

which can be further simplified [28] if one uses the following identity for the fermionic propagators:

$$P_F(z,0;\tau,s)^2 = -\partial_z^2 \ln\vartheta_1(z|\tau) + \partial_v^2 \frac{\vartheta_s(v|\tau)}{\vartheta_s(0|\tau)}\Big|_{v=0}. \tag{2.18}$$

The first term does not depend on the spin structure and can therefore be dropped when dealing with supersymmetric backgrounds. On the other hand, the second term does not depend on z and therefore can be pulled in front of the worldsheet integration in (2.17). The remaining contribution from the bosonic propagators was shown to be [37]

$$\int d^2 z_1 \int d^2 z_2 \left\{ \partial X \partial X (\tilde\psi\tilde\psi)_s^2 - \overline{\partial} X \partial X (\psi\tilde\psi)_s^2 + \overline{\partial} X \overline{\partial} X (\psi\psi)_s^2 - \partial X \overline{\partial} X (\tilde\psi\psi)_s^2 \right\} = \frac{\pi\tau_2}{4}, \tag{2.19}$$

where τ_2 is the imaginary part of the complex modulus of the covering torus. Altogether, using (2.5) the contribution to the induced Planck mass from the orientifold sector becomes $\delta = \delta_{\mathcal{A}} + \delta_{\mathcal{K}} + \delta_{\mathcal{M}}$ where the individual terms are given by

$$\delta_{\mathcal{A}} = \frac{g_s^2}{V_4} \partial_v^2 \sum_{s=2,3,4} \int_0^\infty \frac{dt}{2t} Z_{\mathcal{A}}(t,s) \frac{\vartheta_s(v|it/2)}{\vartheta_s(0|it/2)} \times \frac{\pi t}{8} \Big|_{v=0} \tag{2.20}$$

$$\delta_{\mathcal{K}} = \frac{g_s^2}{V_4} \partial_v^2 \sum_{s=2,3,4} \int_0^\infty \frac{dt}{2t} Z_{\mathcal{K}}(t,s) \frac{\vartheta_s(v|2it)}{\vartheta_s(0|2it)} \times \frac{\pi t}{2} \Big|_{v=0} \tag{2.21}$$

$$\delta_{\mathcal{M}} = \frac{g_s^2}{V_4} \partial_v^2 \sum_{s=2,3,4} \int_0^\infty \frac{dt}{2t} Z_{\mathcal{M}}(t,s) \frac{\vartheta_s(v|1/2+it/2)}{\vartheta_s(0|1/2+it/2)} \times \frac{\pi t}{8} \Big|_{v=0}. \tag{2.22}$$

In order to make potential UV-divergencies more visible, we convert these expressions into tree channel using the following modular transformations

$$\mathcal{A}: \quad \tau = \frac{it}{2} \to -\frac{1}{\tau} = il \tag{2.23}$$

$$\mathcal{K}: \quad \tau = 2it \to -\frac{1}{\tau} = il \tag{2.24}$$

$$\mathcal{M}: \quad \tau = \frac{1}{2} + i\frac{t}{2} \to -\frac{1}{\tau} \to -\frac{1}{\tau} + 2 \to \left(\frac{1}{\tau} - 2\right)^{-1} = -\frac{1}{2} + \frac{i}{2t} = il - \frac{1}{2}. \tag{2.25}$$

Then, using the modular transformation properties (A.8) to (A.13) of Jacobi's theta functions, the individual contributions become

$$\delta_{\mathcal{A}} = \frac{g_s^2}{V_4}\partial_v^2 \sum_{s=2,3,4} \int_0^\infty d\ell \tilde{Z}_{\mathcal{A}}(\ell, s) \frac{\vartheta_s(-iv\ell|i\ell)}{\vartheta_s(0|i\ell)} \frac{\pi}{4\ell} \Big|_{v=0} \tag{2.26}$$

$$\delta_{\mathcal{K}} = \frac{g_s^2}{V_4}\partial_v^2 \sum_{s=2,3,4} \int_0^\infty d\ell \tilde{Z}_{\mathcal{K}}(\ell, s) \frac{\vartheta_s(-iv\ell|i\ell)}{\vartheta_s(0|i\ell)} \frac{\pi}{4\ell} \Big|_{v=0} \tag{2.27}$$

$$\delta_{\mathcal{M}} = \frac{g_s^2}{V_4}\partial_v^2 \sum_{s=2,3,4} \int_0^\infty d\ell \tilde{Z}_{\mathcal{M}}(\ell, s) \frac{\vartheta_s(-2iv\ell| - 1/2 + i\ell)}{\vartheta_s(0| - 1/2 + i\ell)} \frac{\pi}{16\ell} \Big|_{v=0}, \tag{2.28}$$

where $\sum_s \int d\ell \tilde{Z}(\ell, s)$ is the vacuum amplitude after transformation to tree channel. Here, s denotes the spin structure in the tree channel, whose relation to the loop channel spin structure depends on the surface in question. Finally, this simplifies to the following master equations:

$$\delta_{\mathcal{A}} = -\frac{\pi g_s^2}{4V_4}\partial_v^2 \sum_{s=2,3,4} \int_0^\infty d\ell \ell \tilde{Z}_{\mathcal{A}}(\ell, s) \frac{\vartheta_s(v|i\ell)}{\vartheta_s(0|i\ell)} \Big|_{v=0} \tag{2.29}$$

$$\delta_{\mathcal{K}} = -\frac{\pi g_s^2}{4V_4}\partial_v^2 \sum_{s=2,3,4} \int_0^\infty d\ell \ell \tilde{Z}_{\mathcal{K}}(\ell, s) \frac{\vartheta_s(v|i\ell)}{\vartheta_s(0|i\ell)} \Big|_{v=0} \tag{2.30}$$

$$\delta_{\mathcal{M}} = -\frac{\pi g_s^2}{4V_4}\partial_v^2 \sum_{s=2,3,4} \int_0^\infty d\ell \ell \tilde{Z}_{\mathcal{M}}(\ell, s) \frac{\vartheta_s(v| - 1/2 + i\ell)}{\vartheta_s(0| - 1/2 + i\ell)} \Big|_{v=0}. \tag{2.31}$$

All three contributions have the same numerical prefactor. This raises the hope that tadpole cancellation would imply a UV-finite one-loop correction of the Plank mass as was indeed the case for the standard orientifolds in [28]. However, it turns out that UV-finiteness is much less obvious when it comes to orientifolds with intersecting branes.

3. Orientifolds with local tadpole cancellation

In supersymmetric four-dimensional orientifolds with intersecting branes [2, 31, 32], the transverse space is compactified on a Calabi-Yau 3-fold, on which an appropriate orientifold projection acts. In the limit where the Calabi-Yau becomes a toroidal orbifold, we can perform explicit string computations. The orientifold projection combines world-sheet parity with an antiholomorphic involution of the orbifold space. The corresponding Klein bottle tadpoles can be cancelled if one adds intersecting D6-branes. This cancellation can be achieved either locally or globally. Configurations with global tadpole cancellation (i.e. featuring branes which are not fully aligned to the orientifold planes) have been particularly successful in constructing phenomenologically appealing models. Here, we will focus on models with local tadpole cancellation which place intersecting branes on top of the orientifold planes [30]–[32]. The geometrical setup is the following: We consider a factorizable six-torus $T^6 = T^2 \times T^2 \times T^2$ and an orbifold group $\mathbb{Z}_N = \{\Theta^n | n = 1, \ldots, N\}$ where Θ acts on the torus coordinates as

$$\Theta : z_i \mapsto e^{2\pi i v_i} z_i \,, \qquad i = 1, 2, 3 \,. \tag{3.1}$$

The condition for preserved supersymmetry can be written $\sum_i v_i = 0 \bmod 2\pi$. The orientifold projection is given by $P = \frac{1}{2}(1 + \Omega\mathcal{R})$ where $\mathcal{R}$ acts as complex conjugation on all three two-tori. The shape of the tori is essentially fixed by the requirement that the orbifold group should act crystallographically. However, there are two possible alignments of each torus with respect to the $\Omega\mathcal{R}$ orientifold planes. These are known as **A**- and **B**-tori respectively and play an important role in ensuring worldsheet consistency at one-loop level [30, 32].

3.1 The $\mathbb{Z}_3$ orientifold

The simplest non-trivial example of $\mathbb{Z}_N$-orientifolds is the $\mathbb{Z}_3$-model which was first described in [31, 35]. It has a twist vector $v = (1, 1, -2)/3$ and M branes on top of each orientifold plane. Orientifolds of type $\Omega\mathcal{R}$ don't have twisted tadpoles and as a consequence, the orbifold acts on the Chan-Paton indices through traceless matrices,

$$\mathrm{Tr}\, \gamma_k = 0 \,, \qquad k = 1 \ldots N - 1 \,. \tag{3.2}$$

All possible choices for the three tori are allowed. For simplicity, we will choose the **AAA**-lattice. The only difference for a $\mathbf{A}^i\mathbf{B}^{3-i}$ torus would be an overall factor of 3^i in the partition function coming from additional brane intersections and $\mathcal{R}$-invariant orbifold fixed points as well as from different momentum lattices [38, 31]. The various contributions from the annulus, Klein bottle and Möbius strip to the partition function (in the tree channel) are given by:

$$\begin{aligned}
\mathcal{Z}_{\mathcal{A}} &= \frac{M^2}{12\sqrt{3}} \frac{V_4}{(16\pi^2)^2} \sum_{s=2,3,4} (-)^{s-1} \int_0^\infty d\ell \frac{\vartheta_s(0|i\ell)}{\eta(i\ell)^3} \times \\
&\quad \times \left\{ \frac{\vartheta_s(0|i\ell)^3}{\eta(i\ell)^9} \mathcal{L}_{\mathcal{A}}(\ell) + 3\sqrt{3} \prod_{i=1}^{3} \frac{\vartheta_s(v_i|i\ell)}{\vartheta_1(v_i|i\ell)} - 3\sqrt{3} \prod_{i=1}^{3} \frac{\vartheta_s(2v_i|i\ell)}{\vartheta_1(2v_i|i\ell)} \right\}
\end{aligned} \tag{3.3}$$

$$\begin{aligned}
\mathcal{Z}_{\mathcal{K}} &= \frac{16}{12\sqrt{3}} \frac{V_4}{(16\pi^2)^2} \sum_{s=2,3,4} (-)^{s-1} \int_0^\infty d\ell \frac{\vartheta_s(0|i\ell)}{\eta(i\ell)^3} \times \\
&\quad \times \left\{ \frac{\vartheta_s(0|i\ell)^3}{\eta(i\ell)^9} \mathcal{L}_{\mathcal{K}}(\ell) + 3\sqrt{3} \prod_{i=1}^{3} \frac{\vartheta_s(v_i|i\ell)}{\vartheta_1(v_i|i\ell)} - 3\sqrt{3} \prod_{i=1}^{3} \frac{\vartheta_s(2v_i|i\ell)}{\vartheta_1(2v_i|i\ell)} \right\}
\end{aligned} \tag{3.4}$$

$$
\begin{aligned}
\mathcal{Z}_{\mathcal{M}} = & -\frac{8M}{12\sqrt{3}}\frac{V_4}{(16\pi^2)^2}\sum_{s=2,3,4}(-)^{s-1}\int_0^\infty d\ell \frac{\vartheta_s(0|-1/2+i\ell)}{\eta(i\ell)^3}\times \\
& \times\left\{\frac{\vartheta_s(0|-1/2+i\ell)^3}{\eta(-1/2+i\ell)^9}\mathcal{L}_{\mathcal{M}}(\ell)+3\sqrt{3}\prod_{i=1}^{3}\frac{\vartheta_s(v_i|-1/2+i\ell)}{\vartheta_1(v_i|-1/2+i\ell)}\right. \\
& \left. -3\sqrt{3}\prod_{i=1}^{3}\frac{\vartheta_s(2v_i|-1/2+i\ell)}{\vartheta_1(2v_i|-1/2+i\ell)}\right\}. \qquad (3.5)
\end{aligned}
$$

Here, $\mathcal{L}_\sigma(\ell)$ denotes lattice contributions (from summing over Kaluza-Klein and winding modes) which rapidly tend to unity as l goes to infinity and whose exact form we won't need. Tadpole cancellation therefore requires $M = 4$. Knowing the different contributions to the partition function, we can now compute the induced Planck mass. Using the tadpole cancellation condition, $M = 4$, as well as equations (2.29) to (2.31) and a variation of the Jacobi identity (A.21), (A.22), we find:

$$
\delta_{\mathcal{A}} = +\frac{1}{2^7\pi^2}\int_0^\infty d\ell\ell\sum_{i=1}^{3}\frac{\vartheta_1'(v_i|i\ell)}{\vartheta_1(v_i|i\ell)} \qquad (3.6)
$$

$$
\delta_{\mathcal{K}} = +\frac{1}{2^7\pi^2}\int_0^\infty d\ell\ell\sum_{i=1}^{3}\frac{\vartheta_1'(v_i|i\ell)}{\vartheta_1(v_i|i\ell)} \qquad (3.7)
$$

$$
\delta_{\mathcal{M}} = -\frac{2}{2^7\pi^2}\int_0^\infty d\ell\ell\sum_{i=1}^{3}\frac{\vartheta_1'(v_i|-1/2+i\ell)}{\vartheta_1(v_i|-1/2+i\ell)}. \qquad (3.8)
$$

Note that the four-dimensional volume V_4 drops out as required. Adding up these contributions gives the simple expression

$$
\delta = \frac{1}{2^6\pi^2}\int_0^\infty d\ell\ell\sum_{i=1}^{3}\left(\frac{\vartheta_1'(v_i|i\ell)}{\vartheta_1(v_i|i\ell)}-\frac{\vartheta_1'(v_i|-1/2+i\ell)}{\vartheta_1(v_i|-1/2+i\ell)}\right). \qquad (3.9)
$$

One immediately sees that the integral is UV-finite because in the limit $l \to \infty$ the two different expressions in the bracket rapidly approach the same constant. However, one can even compute the integral explicitly, using special properties of the Jacobi functions. The computation is carried out in appendix B, leading to the final result (reinserting a factor of $1/\alpha' = M_s^2$):

$$
\delta = \frac{1}{2^{12}\pi^2\sqrt{3}}\left(\zeta\left(2,\frac{5}{6}\right)-\zeta\left(2,\frac{1}{6}\right)\right)g_s^2M_s^2 \approx 5.0\times 10^{-4}g_s^2M_s^2\,. \qquad (3.10)
$$

Here, ζ is the generalized or Hurwitz zeta function. Thus, we get a finite, non-vanishing one-loop contribution to the induced four-dimensional Planck mass. As could have been expected from the localization of the relevant interactions, the induced Planck mass is independent of the compactification scale. It is also roughly of the order of the string mass. The remaining orientifolds which were described in [31], namely the $\mathbb{Z}_4$-, $\mathbb{Z}_6$- and $\mathbb{Z}_6'$-models can be treated in the same manner. There is, however, one modification coming from the fact that these models contain a $\mathbb{Z}_2$ subgroup.

3.2 Even order $\mathbb{Z}_N$-orientifolds

If ω is the generator of a $\mathbb{Z}_2$ subgroup, then its twist vector necessarily has a zero in one component, i.e. one torus is invariant under the action of ω. As a consequence, the corresponding contribution to the vacuum loop amplitude contains a factor of ϑ_1^2/η^6 and a non-trivial lattice factor from Kaluza-Klein and winding modes. As an illustration of this point, we write down the computation of the induced Planck mass for the $\mathbb{Z}_4$-case with type **ABA** torus which has a twist vector $v = (1/4, 1/4, -1/2)$. The $\mathcal{A}$, $\mathcal{K}$ and $\mathcal{M}$ vacuum amplitudes were calculated in [31]. In tree channel, they are given by:

$$\begin{aligned}\mathcal{Z}_{\mathcal{A}} = \frac{M^2}{16}\frac{V_4}{(16\pi^2)^2}\sum_{s=1}^{4}(-)^{s-1}\int_0^\infty d\ell \frac{\vartheta_s(0|i\ell)}{\eta(i\ell)^3} \times \\ \times \left\{\frac{\vartheta_s(0|i\ell)^3}{\eta(i\ell)^9}\mathcal{L}_{\mathcal{A}}(\ell) - \frac{\vartheta_s(0|i\ell)}{\eta(i\ell)^3}\frac{\vartheta_s(1/2|i\ell)^2}{\vartheta_1(1/2|i\ell)^2}\tilde{\mathcal{L}}[1,1](\ell) + \right. \\ \left. + 4\prod_{i=1}^{3}\frac{\vartheta_s(v_i|i\ell)}{\vartheta_1(v_i|i\ell)} - 4\prod_{i=1}^{3}\frac{\vartheta_s(3v_i|i\ell)}{\vartheta_1(3v_i|i\ell)}\right\}\end{aligned} \tag{3.11}$$

$$\begin{aligned}\mathcal{Z}_{\mathcal{K}} = 16\frac{V_4}{(16\pi^2)^2}\sum_{s=1}^{4}(-)^{s-1}\int_0^\infty d\ell \frac{\vartheta_s(0|i\ell)}{\eta(i\ell)^3} \times \\ \times \left\{\frac{\vartheta_s(0|i\ell)^3}{\eta(i\ell)^9}\mathcal{L}_{\mathcal{K}}(\ell) - \frac{\vartheta_s(0|il)}{\eta(i\ell)^3}\frac{\vartheta_s(1/2|i\ell)^2}{\vartheta_1(1/2|i\ell)^2}\tilde{\mathcal{L}}[2,2](\ell) + \right. \\ \left. + 4\prod_{i=1}^{3}\frac{\vartheta_s(v_i|i\ell)}{\vartheta_1(v_i|i\ell)} - 4\prod_{i=1}^{3}\frac{\vartheta_s(3v_i|i\ell)}{\vartheta_1(3v_i|i\ell)}\right\}\end{aligned} \tag{3.12}$$

$$\begin{aligned}\mathcal{Z}_{\mathcal{M}} = -2M\frac{V_4}{(16\pi^2)^2}\sum_{s=1}^{4}(-)^{s-1}\int_0^\infty d\ell \frac{\vartheta_s(0|-1/2+i\ell)}{\eta(-1/2+i\ell)^3} \times \\ \times \left\{\frac{\vartheta_s(0|-1/2+i\ell)^3}{\eta(-1/2+i\ell)^9}\mathcal{L}_{\mathcal{M}}(\ell) - \right. \\ - \frac{\vartheta_s(0|-1/2+i\ell)}{\eta(-1/2+i\ell)^3}\frac{\vartheta_s(1/2|-1/2+i\ell)^2}{\vartheta_1(1/2|-1/2+i\ell)^2} \times \\ \times \tilde{\mathcal{L}}[2,2](\ell) + 4\prod_{i=1}^{3}\frac{\vartheta_s(v_i|-1/2+i\ell)}{\vartheta_1(v_i|-1/2+i\ell)} \\ \left. - 4\prod_{i=1}^{3}\frac{\vartheta_s(3v_i|-1/2+i\ell)}{\vartheta_1(3v_i|-1/2+i\ell)}\right\}.\end{aligned} \tag{3.13}$$

Again, the precise form of the lattice factors $\mathcal{L}_\sigma$ coming from untwisted fields in the loop channel will be irrelevant. On the other hand, the lattice factors coming from $\mathbb{Z}_2$ twisted strings in the loop channel are relevant. Their explicit form is given by

$$\tilde{\mathcal{L}}[\alpha,\beta](\ell) = \left(\sum_{m\in\mathbb{Z}} e^{-\alpha\pi\ell m^2 R^2/2}\right)\left(\sum_{n\in\mathbb{Z}} e^{-2\beta\pi\ell n^2/R^2}\right), \tag{3.14}$$

where R is the compactification radius (for simplicity, all six radii are taken to be equal). Tadpole cancellation requires $M = 16$. Then, using equations (A.16), (A.22) and (A.23), we find

$$\delta_{\mathcal{A}} = +\frac{g_s^2}{8\pi^2}\left[\int_0^\infty d\ell\ell\sum_{i=1}^3 \frac{\vartheta'_1(v_i|i\ell)}{\vartheta_1(v_i|i\ell)} + \frac{\pi}{4}\int_0^\infty d\ell\ell\mathcal{L}[1,1](\ell)\right] \tag{3.15}$$

$$\delta_{\mathcal{K}} = +\frac{g_s^2}{8\pi^2}\left[\int_0^\infty d\ell\ell\sum_{i=1}^3 \frac{\vartheta'_1(v_i|i\ell)}{\vartheta_1(v_i|i\ell)} + \frac{\pi}{4}\int_0^\infty d\ell\ell\mathcal{L}[2,2](\ell)\right] \tag{3.16}$$

$$\delta_{\mathcal{M}} = -\frac{g_s^2}{4\pi^2}\left[\int_0^\infty d\ell\ell\sum_{i=1}^3 \frac{\vartheta'_1(v_i|-1/2+i\ell)}{\vartheta_1(v_i|-1/2+i\ell)} + \frac{\pi}{4}\int_0^\infty d\ell\ell\mathcal{L}[2,2](\ell)\right], \tag{3.17}$$

and therefore, the induced Planck mass becomes

$$\begin{aligned}\delta = \frac{g_s^2}{4\pi^2}\int_0^\infty d\ell\ell\sum_{i=1}^3\left(\frac{\vartheta'_1(v_i|i\ell)}{\vartheta_1(v_i|i\ell)} - \frac{\vartheta'_1(v_i|-1/2+i\ell)}{\vartheta_1(v_i|-1/2+i\ell)}\right) + \\ + \frac{g_s^2}{32\pi}\int_0^\infty d\ell\ell\left(\mathcal{L}[1,1](\ell) - \mathcal{L}[2,2](\ell)\right).\end{aligned} \tag{3.18}$$

The first term comes from the untwisted sector and is of the same form as the corresponding term for the $\mathbb{Z}_3$-model. Evaluating the integral gives a contribution of the form (again, reinserting a factor of $2/\alpha'$):

$$\delta_{\text{untwisted}} = \frac{1}{2^8\pi^2}\left((\zeta(2,1/4) - \zeta(2,3/4)\right) g_s^2 M_s^2 \approx 5.8\times 10^{-3} g_s^2 M_s^2\,. \tag{3.19}$$

On the other hand, the second term is derived from the $\mathbb{Z}_2$-twisted sector. In this case, evaluating the integral gives the following expression (see appendix B):

$$\delta_{\text{twisted}} = \frac{3}{32\pi^3}\left(\sum_{n,m=1}^\infty \frac{1}{\left(\frac{1}{2}(mR)^2 + 2\left(\frac{n}{R}\right)^2\right)^2} + \frac{\pi^4}{180}\left(\frac{R^2}{2} + \frac{2}{R^2}\right)\right) M_s^2\,. \tag{3.20}$$

Because the $\mathbb{Z}_2$-subgroup of the orbifold group leaves the third torus invariant, the $R \to \frac{\alpha'}{R}$ T-duality reappears in the $\mathbb{Z}_2$-twisted sector. For R much bigger than one, the term which is proportional to R^2 dominates over all other contributions to the induced Planck mass. We can write:

$$\delta = \frac{\pi}{2^6\cdot 3\cdot 5} R^2 g_s^2 M_s^4 + \mathcal{O}(R^0)\,. \tag{3.21}$$

We recognize a Kaluza-Klein like dependence on the compactification scale. In fact, the $\mathbb{Z}_2$-twisted sector consists of open and closed string states which are localized on a six-dimensional surface. We can interpret the above contribution to the four-dimensional Planck mass as coming from an induced six-dimensional Planck mass which is subsequently reduced to four dimensions by a Kaluza-Klein mechanism.

4. Conclusions and discussion

Quantum loop corrections generically renormalize the four-dimensional Planck mass in brane world models. We have shown how these corrections arise in supersymmetric orientifold models with intersecting branes. For $\Omega\mathcal{R}$-orientifolds with local tadpole cancellation, we found that tadpole cancellation implies a UV-finite graviton scattering amplitude and we have explicitly computed the induced four-dimensional Planck mass in this case. We would like to stress that, in principle, computing the induced Planck mass is completely under control for all supersymmetric Intersecting Brane Worlds. One can use the formuli (2.29) to (2.31) as a black box taking only the open string partition function as an input. However, UV-divergencies in graviton scattering amplitudes could potentially spoil perturbative calculability. It remains an open question as to whether this happens in the context of graviton scattering in the presence of D-branes and O-planes.

DGP-gravity allows us in principle to think about non-compact extra dimensions. However, one of the most attractive features of intersecting brane worlds is that for compactified extra dimensions, two branes can multiply intersect with each other, giving rise to family replication. Also, branes wrapping infinite volume cycles in the extra dimensions don't support local gauge symmetries as their four-dimensional coupling constant is proportional to the branes' volume in the extra dimensions. If we stick to compact extra dimensions, the four-dimensional Planck mass gets a Kaluza-Klein contribution from the ten-dimensional Planck mass which is of the form

$$M_{P,KK}^2 \sim M_s^8 V_6 \,, \tag{4.1}$$

where V_6 is the compactification volume. For supersymmetric string vacua, it is natural to take the string scale to be of the order of the GUT scale and as a consequence, the Kaluza-Klein mechanism accounts for the observed Planck mass if the extra dimensions are moderately enlarged. Induced gravity doesn't change the picture significantly. However, there are at least two reasons why DGP-gravity might be important in intersecting brane worlds. First, a bottom-up approach to intersecting brane worlds has been proposed [39] in which branes wrap small cycles in a non-compact Calabi-Yau 3-fold. There, an induced Einstein-Hilbert term on the brane intersections would localize gravity and explain the observed $1/r^2$-dependence of the gravitational force. Second, and perhaps more importantly, one of the most exciting prospects of future table-top experiments is the possibility of finding large extra dimensions. In non-supersymmetric models without induced gravity, these could well be in the sub-millimeter regime. Constraints on the size of the extra dimensions are derived from the energy loss that high energy events on the brane, such as supernovae or collider experiments, would suffer due to emission of light Kaluza-Klein modes of the graviton into the bulk. Because in DGP-gravity, graviton emission is strongly suppressed for energies greater than a cross-over scale, which is controlled by the magnitude of the induced Planck mass and the effective width of the brane, the constraints on the size of extra dimensions can be relaxed considerably [40, 4]. While this argument is not relevant for toroidal orientifolds with intersecting $D6$-branes,[4] it applies to more general backgrounds with large transverse directions and to orientifolds with intersecting $D4$-branes or $D5$-branes.

The specific models we examined in this work are phenomenologically unappealing, being non-chiral. However, we regard our results on induced gravity as simple examples of a more general mechanism. In this respect it is interesting that it is possible to get an induced Planck mass which grows linearly with the volume of some extra dimensions. This means that for large extra dimensions, the pure Kaluza-Klein contribution to the four-dimensional Planck mass does not necessarily dominate over the induced Einstein-Hilbert term.

As we pointed out, it is not clear how tadpole cancellation insures UV-finiteness of the graviton scattering amplitude in more general backgrounds. In general, the divergencies in the one-loop graviton amplitude have a very different structure from those in the vacuum amplitude. For example, two D6-branes which intersect at angles $\phi_i, i = 1\dots3$, lead to a term of the form:

$$\frac{\vartheta_s(0)}{\eta^3}\prod_{i=1}^{3}\frac{\vartheta_s(\phi_i/\pi)}{\vartheta_1(\phi_i/\pi)} \tag{4.2}$$

in the tree level partition function. As can easily be checked, this term contributes to the RR-tadpole divergence in the vacuum amplitude in the following form:

$$\sim\prod_{i=1}^{3}\cot(\phi_i)\int_0^\infty d\ell\,. \tag{4.3}$$

On the other hand, it contributes to the divergence in the graviton scattering amplitude through a term

$$\sim\sum_{i=1}^{3}\cot(\phi_i)\int_0^\infty d\ell\ell\,. \tag{4.4}$$

If we consider orientifold models where D-branes are not aligned to the O-planes, the partition functions in the $\mathcal{A}$-, $\mathcal{K}$- and $\mathcal{M}$-sector don't have a mutually identical structure any longer. Instead, they conspire non-trivially to cancel the infinities in the one-loop vacuum amplitude. It is therefore not obvious whether UV-finiteness holds for the graviton scattering amplitude. In the absence of any general theorems about finiteness of open string scattering amplitudes, it would be interesting to have some empirical evidence instead. For a few simple examples, we have checked that the graviton scattering amplitude remains finite by an explicit computation. However, we are unable at the moment to give a general argument in favour of finiteness. We hope to report on this issue in the future.

Finally, we would like to point out that an interesting extension of our work would be to compute moduli scattering in toroidal $\mathcal{N} = 1$ string compactifications and thus determine one-loop corrections to the Kähler potential. A deviation from the no-scale form would be highly interesting in the context of moduli stabilization.

[4] For intersecting D6-branes wrapping T^6, the maximum size of the extra dimensions is more seriously constrained by the non-observation of Kaluza-Klein modes of the Standard model gauge fields as there is no direction which is transverse to all branes

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A. Jacobi Theta Functions

The Jacobi Theta functions are defined by

$$\vartheta_1(z|\tau) = \sum_{n\in\mathbb{Z}} q^{(n+1/2)^2/2} e^{2\pi i(n+1/2)(z+1/2)}\,, \qquad \vartheta_2(z|\tau) = \sum_{n\in\mathbb{Z}} q^{(n+1/2)^2/2} e^{2\pi i(n+1/2)z} \tag{A.1}$$

$$\vartheta_3(z|\tau) = \sum_{n\in\mathbb{Z}} q^{n^2/2} e^{2\pi i n z}\,, \qquad \vartheta_4(z|\tau) = \sum_{n\in\mathbb{Z}} q^{n^2/2} e^{2\pi i n z+1/2)}\,, \tag{A.2}$$

where $q = e^{2\pi i\tau}$ is called the nome and the multi-valued function q^{λ} is interpreted to stand for $e^{2\pi i\tau\lambda}$. They have alternative product representations

$$\vartheta_1(z|\tau) = i\eta(\tau) e^{\pi i z} q^{\frac{1}{12}} \prod_{n=1}^{\infty} \left(1 - e^{2\pi i z} q^n\right)\left(1 - e^{-2\pi i z} q^{n-1}\right) \tag{A.3}$$

$$\vartheta_2(z|\tau) = \eta(\tau) e^{\pi i z} q^{\frac{1}{12}} \prod_{n=1}^{\infty} \left(1 + e^{2\pi i z} q^n\right)\left(1 + e^{-2\pi i z} q^{n-1}\right) \tag{A.4}$$

$$\vartheta_3(z|\tau) = \eta(\tau) q^{-\frac{1}{24}} \prod_{n=1}^{\infty} \left(1 + e^{2\pi i z} q^{n-\frac{1}{2}}\right)\left(1 + e^{-2\pi i z} q^{n-\frac{1}{2}}\right) \tag{A.5}$$

$$\vartheta_4(z|\tau) = \eta(\tau) q^{-\frac{1}{24}} \prod_{n=1}^{\infty} \left(1 - e^{2\pi i z} q^{n-\frac{1}{2}}\right)\left(1 - e^{-2\pi i z} q^{n-\frac{1}{2}}\right), \tag{A.6}$$

where η is the Dedekind function

$$\eta(\tau) = q^{1/24} \prod_{n=1}^{\infty} (1 - q^n)\,. \tag{A.7}$$

By convention, the derivative is taken with respect to the first variable. The following modular transformations hold:

$$\vartheta_1(z|\tau) = (-i\tau)^{-1/2} \exp\left(-\frac{i\pi z^2}{\tau}\right) i\vartheta_1\left(\frac{z}{\tau}\middle| -\frac{1}{\tau}\right) \tag{A.8}$$

$$\vartheta_2(z|\tau) = (-i\tau)^{-1/2} \exp\left(-\frac{i\pi z^2}{\tau}\right) \vartheta_4\left(\frac{z}{\tau}\middle| -\frac{1}{\tau}\right) \tag{A.9}$$

$$\vartheta_3(z|\tau) = (-i\tau)^{-1/2} \exp\left(-\frac{i\pi z^2}{\tau}\right) \vartheta_3\left(\frac{z}{\tau}\middle| -\frac{1}{\tau}\right) \tag{A.10}$$

$$\vartheta_4(z|\tau) = (-i\tau)^{-1/2} \exp\left(-\frac{i\pi z^2}{\tau}\right) \vartheta_2\left(\frac{z}{\tau}\middle| -\frac{1}{\tau}\right) \tag{A.11}$$

for S-transformations and

$$\vartheta_1(z|\tau) = e^{i\pi/4}\vartheta_1(z|\tau+1)\,; \qquad \vartheta_2(z|\tau) = e^{i\pi/4}\vartheta_2(z|\tau+1) \tag{A.12}$$

$$\vartheta_3(z|\tau) = \vartheta_4(z|\tau+1)\,; \qquad \vartheta_4(z|\tau) = \vartheta_3(z|\tau+1) \tag{A.13}$$

for T-transformations. The Dedekind function obeys

$$\eta(\tau) = (-i\tau)^{-1/2}\eta\left(-\frac{1}{\tau}\right)\,; \qquad \eta(\tau) = e^{-i\pi/12}\eta(\tau+1)\,. \tag{A.14}$$

The ratios of derivatives of Jacobi theta functions to the functions themselves have a simple form [41]. In particular, we have:

$$\frac{\vartheta_1'(z|\tau)}{\vartheta_1(z|\tau)} = \cot(\pi z) + 4\sum_{n=1}^{\infty}\frac{q^n}{1-q^n}\sin(2\pi n z)\,. \tag{A.15}$$

The first theta function also satisfies the following equations:

$$\vartheta_1(0|\tau) = 0\,, \qquad \vartheta_1'(0|\tau) = -2\pi\eta(\tau)^3\,, \qquad \vartheta_1''(0|\tau) = 0\,. \tag{A.16}$$

The Jacobi theta functions are solutions to the heat equation

$$\partial_z^2\vartheta_s(z|\tau) = 4\pi i\partial_\tau\vartheta_s(z|\tau)\,. \tag{A.17}$$

They satisfy a great number of identities which can be viewed as generalizations of Jacobi's 'abstruse identity'. Most useful to us is the following:

$$\sum_{s=1}^{4}(-)^{s-1}\prod_{i=1}^{4}\vartheta_s(v_i) = 2\prod_{i=1}^{4}\vartheta_1(v_i')\,, \tag{A.18}$$

where one defines

$$v_1' = \frac{1}{2}(v_1+v_2+v_3+v_4)\,; \qquad v_2' = \frac{1}{2}(v_1+v_2-v_3-v_4) \tag{A.19}$$

$$v_3' = \frac{1}{2}(v_1-v_2+v_3-v_4)\,; \qquad v_4' = \frac{1}{2}(v_1-v_2-v_3+v_4)\,. \tag{A.20}$$

and we omit the second argument of the theta functions. For the special case of $v_1+v_2+v_3 = 0$, we get the following relation:

$$\sum_{s=2,3,4}(-)^{s-1}\left\{\vartheta_s(v)\prod_{i=1}^{3}\vartheta_s(v_i)\right\} = -\vartheta_1(v)\prod_{i=1}^{3}\vartheta_1(v_i) + 2\vartheta_1\left(\frac{v}{2}\right)\prod_{i=1}^{3}\vartheta_1\left(\frac{v}{2}+v_i\right)\,. \tag{A.21}$$

In particular, using equations (A.16) and the antisymmetry of $\theta_1(z|\tau)$ about $z=0$, we find:

$$\partial_v^2\sum_{s=2,3,4}(-)^{s-1}\left\{\vartheta_s(v)\prod_{i=1}^{3}\vartheta_s(v_i)\right\}\Bigg|_{v=0} = -\pi\eta(\tau)^3\sum_{i=1}^{3}\frac{\vartheta_1'(v_i)}{\vartheta_1(v_i)}\prod_{j=1}^{3}\vartheta_1(v_j)\,. \tag{A.22}$$

This expression is well-defined if none of the v_i vanish individually. On the other hand, we find that for $v_1 = v_2+v_3 = 0$, the following equation holds:

$$\partial_v^2\sum_{s=2,3,4}(-)^{s-1}\left\{\vartheta_s(v)\prod_{i=1}^{3}\vartheta_s(v_i)\right\}\Bigg|_{v=0} = 2\pi^2\eta(\tau)^6\prod_{j=2,3}\vartheta_1(v_j)\,. \tag{A.23}$$

B. Some useful integrals

In this appendix, we evaluate two integrals which appear in the computation of the graviton scattering amplitude. The first integral is given by

$$I_1(z) = \int_0^\infty d\ell\, \ell \left(\frac{\vartheta_1'(z|\tau)}{\vartheta_1(z|\tau)} - \frac{\vartheta_1'(z| -1/2+\tau)}{\vartheta_1(z|-1/2+\tau)} \right). \tag{B.1}$$

If we define $q \equiv \exp(2\pi\tau)$, we can write (using (A.15)):

$$I_1 = \int_0^\infty d\ell\, \ell \sum_{n=1}^\infty \sin(2\pi n z) \left(\frac{q^n}{1-q^n} - \frac{(-q)^n}{1-(-q)^n} \right) \tag{B.2}$$

$$= \frac{1}{2\pi^2} \sum_{n \text{ odd}} \frac{\sin(2\pi n z)}{n^2} \int_0^\infty d\ell\, \ell \frac{e^{-\ell}}{1-e^{-2\ell}} \tag{B.3}$$

$$= \frac{1}{16} \sum_{n \text{ odd}} \frac{\sin(2\pi n z)}{n^2}. \tag{B.4}$$

This expression can be evaluated in terms of a finite sum of Hurwitz zeta functions,

$$\zeta(s,a) = \sum_{n=0}^\infty \frac{1}{(n+a)^s}, \tag{B.5}$$

because the integral in question always appears with rational argument. For instance, in the $\mathbb{Z}_3$- and $\mathbb{Z}_6$-models we have to compute

$$I_1\left(\frac{1}{3}\right) = \frac{1}{16} \sum_{n \text{ odd}} \frac{\sin(2\pi n/3)}{n^2} \tag{B.6}$$

$$= \frac{1}{16} \sin\left(\frac{2\pi}{3}\right) \sum_{n=0}^\infty \left(\frac{1}{(6n+1)^2} - \frac{1}{(6n+5)^2} \right) \tag{B.7}$$

$$= \frac{\sqrt{3}}{32 \cdot 36} \left(\zeta\left(2, \frac{1}{6}\right) - \zeta\left(2, \frac{5}{6}\right) \right). \tag{B.8}$$

The second integral we are interested in appears whenever the orbifold group has a $\mathbb{Z}_2$ subgroup. It is given by

$$I_2(a,R) = \int_0^\infty d\ell\, \ell \sum_{m,n\in\mathbb{Z}} e^{-a\ell\pi(m^2R^2/2 + 2n^2/R^2)}. \tag{B.9}$$

This integral is divergent due to the $m = n = 0$ part of the sum. However, we can formally separate the divergence which will cancel in the full scattering amplitude and focus on the convergent part of the integral:

$$\begin{aligned}\tilde{I}_2(a,R) &= \int_0^\infty d\ell\, \ell \sum_{\substack{n,m\in\mathbb{Z} \\ (n,m)\neq(0,0)}} e^{-a\ell\pi(m^2R^2/2+2n^2/R^2)} \\ &= \int_0^\infty d\ell\ell \left(4 \sum_{n,m=1}^\infty e^{-a\ell\pi(m^2R^2/2+2n^2/R^2)} + 2 \sum_{m=1}^\infty e^{-a\ell\pi m^2R^2/2} + 2 \sum_{n=1}^\infty e^{-a\ell\pi 2n^2/R^2} \right)\end{aligned}$$

$$= \frac{4}{a^2\pi^2} \sum_{n,m=1}^{\infty} \frac{1}{(m^2R^2/2 + 2(n/R)^2)^2} + \frac{2\zeta(4)}{a^2\pi^2}\left(\frac{R^2}{2} + \frac{2}{R^2}\right). \tag{B.10}$$

It is interesting to consider the case where $R \gg 1$. In this limit, we can approximate the sum over n by an integral, making the following substitutions:

$$\frac{1}{R} = \Delta x, \qquad \frac{n}{R} = x, \qquad \sum_{n=1}^{\infty} \approx R\int_0^{\infty} dx. \tag{B.11}$$

From this, it follows that

$$\sum_{n,m=1}^{\infty} \frac{1}{(m^2R^2/2 + 2(n/R)^2)^2} \approx R \sum_{n,m=1}^{\infty} \int_0^{\infty} \frac{dx}{(m^2R^2/2 + 2x^2)^2} = \frac{\pi}{2R^2}\zeta(3). \tag{B.12}$$

Therefore, we can write:

$$\tilde{I}(a,R) = \frac{\zeta(4)}{a^2\pi^2} R^2 + \mathcal{O}(1/R^2). \tag{B.13}$$

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